Metric Mean Distance Reconsidered

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Keywords
spatial configuration; methodology; modelling; mean distance; metric distance;

Abstract
Metric Mean Distance (MMD) is normally calculated in Space Syntax as a weighted graph measure, i.e. by means of discretization. In this paper, I propose a new way of calculating MMD without discretization. In doing so, I discuss the basic concepts of the proposed algorithm and its analytical and theoretical implications. My approach is based on a dissection of a plan in discrete regions that are formed by a sequence of truncated, non-overlapping isovists. A triangulation of these regions reduces the problem of calculating MMD in any plane polygon to that of calculating the MMD of a corner in a plane triangle from the surface of the triangle. The latter can then be solved by integration.

Subsequent isovists can also be used to generate a map with isodistance lines from a single point. I call this map a Metric Fingerprint because of its resemblance to a fingerprint and because it illustrates the unique metric genius loci of any point location in a plan. I propose a format for drawing Metric Fingerprints that can be used as a technique for spatial analysis. Specifically, the format of metric fingerprints includes the notation of watersheds of shortest paths in plans that are not simply connected. Series of Metric Fingerprints along a continuous path can be played as films in dynamic space-time representation of a metric field. I also suggest how statistical measures can be derived from distributions of MMD values and used for allometric studies and for defining a new scale-invariant value of centrality in plan shapes.

1. Metric and topo-geometric measures

Space Syntax draws a theoretical distinction between two types of spatial analysis. One is the topo-geometric analysis of space, such as VGA or access graph analysis. The other is metric analysis, such as Metric Mean Distance (MMD). Both types of analysis are based on the identification of a finite number of discrete spatial elements whose relation to each other - be it metric, angular, topo-geometric, local, global, or local-to-global, can be analysed with a graph. Where cities are concerned, the distinction between metric and topo-geometric analysis brings with it a distinction of scale. Present research suggests that topo-geometric properties better describe the differentiation of space on a global level, whereas metric properties seem more suited for describing the local differentiation of space on a local level (Hillier 2007).

Up until now, work with Space Syntax has focussed more strongly on topo-geometic analysis. This is because the basic notion of space syntax - according to which spatial structure and social structure emerge concurrently - implies a certain abstraction from metrics and shape. Abstract topo-geometrical models were needed to find a common analytical ground for a spatial theory that also seeks to be a theory of society. What's more, beyond being more theoretically grounded in Space Syntax than metric analysis, topo-geometrical analysis has proven more fruitful analytically, particularly with regard to its impressive results in predicting movement. Nevertheless, researchers in related fields have insisted that Space Syntax underestimates the importance of metrics, and have called for the introduction of metric weighting factors into topo-geometric analysis (Ratti

2004, Montello 2007). Yet this hybridisation of measures has, in the case of cities, produced weak results. Hillier has shown that at best, one can say that metric weighting hasn't harmed topo-geometric analysis. The question therefore is whether and how metric analysis can be developed to improve the space syntax analysis. One direction of research has been suggested by Hillier et al.: MMD with radius restriction (MMDr) could be a model to describe a "natural spatial area-isation" of cities at various scales (Hillier 2007, pp. 001.01). In this paper I propose further directions of research based on a continuous model of MMD.

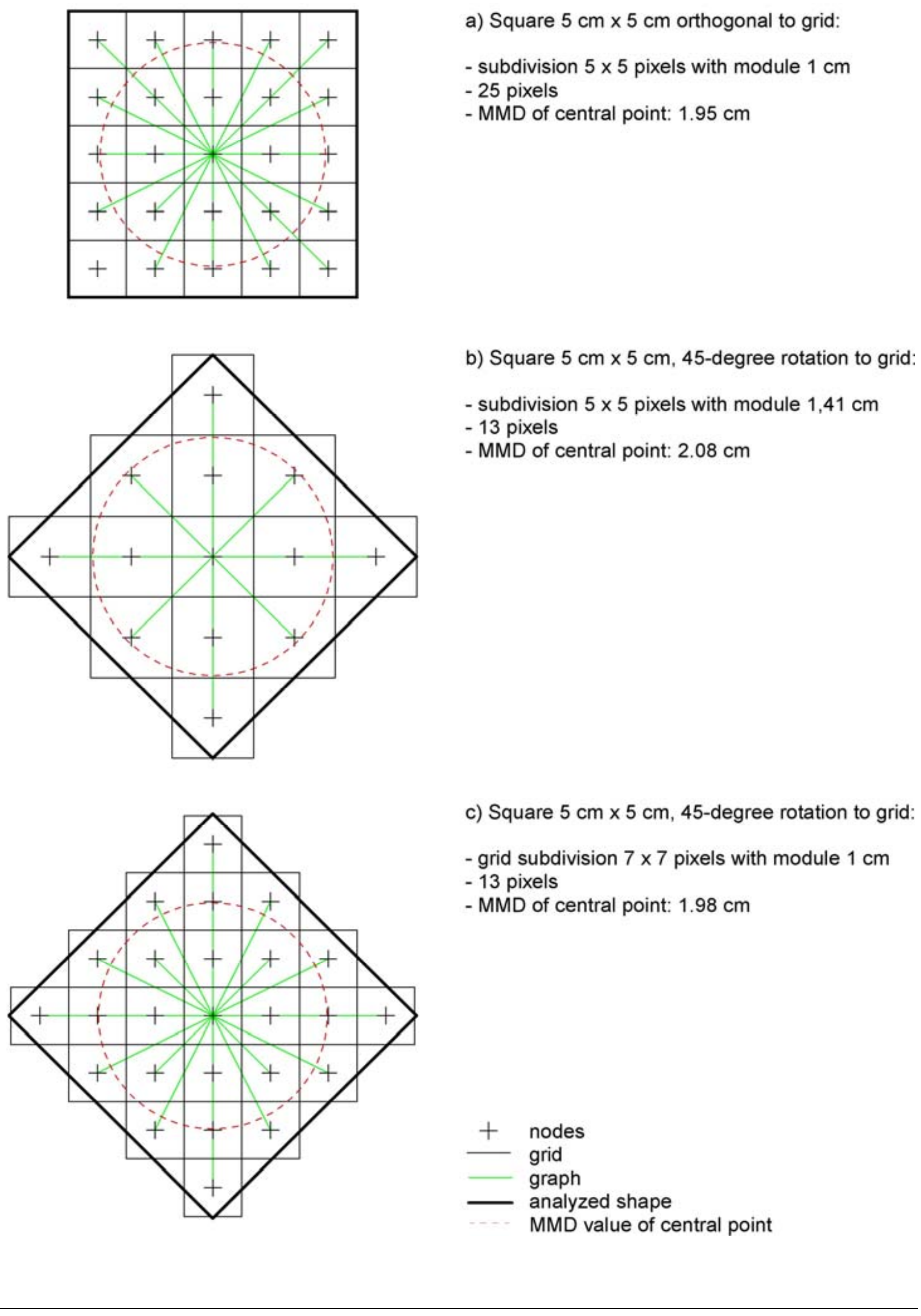


Figure 1
MMD Values Obtained from Different Discretizations of a Square

In the context of Space Syntax, Metric Mean Distance is calculated as an average measure in a weighted graph. In order to construct the graph, a two-dimensional plan is discretized in pixels and the pixels are abstracted as nodes. MMD is the mean distance in meters one pixel has to all other pixels of a plan. The only particularity of the metric mean value is that metric distances are not measured as the crow flies but as the shortest path from pixel to pixel around the angles of their obstacles. This aspect of the model accounts for the fact that nobody can walk through a wall. To simulate the shortest paths in real environments, modelled paths must lead around - not through - a plan's obstacles and angles.

The numeric reliability of this weighted-graph model of MMD greatly depends on the resolution of the pixel grid used for discretization and on the geometrical irregularity of the analysed shapes with respect to the regular pixel grid. A simple example illustrates one of the problems with discretization: A weighted graph from a discretization for a square plan rotated 45° degrees differs significantly from a weighted graph from a discretization for an non-rotated square plan (Figure 1). The differences in MMD values between the rotated and the non-rotated version do disappear with increasing resolution, but increasing resolution costs considerably more computing time, which makes impractical the MMD analysis of high-resolution-models of larger spaces with many angles and holes.

Another problem is that the discrete model of MMD appears conceptually questionable. The graph-based model seems to indicate that the MMD is a value arising from a relationship of a finite set of discrete spatial elements. But the distinguishing feature of metric distances is their continuity, that is to say: their non-discreteness. Since we know that metric space is, at least up to the subatomic level, infinitesimally fine grained, I would like to propose a continuous model for calculating MMD based on the metric field of a single point location within a plan system.

2. Metric Fingerprints

Currently, Space Syntax possesses no convention for generating an analytical map that represents the continuous metric field of a single point location and that could thus be regarded as the metric equivalent of the justified graph or the isovist of a certain location. Yet such a map of a local metric field might be useful to draw, especially in cases where discrete spatial units such as rooms with doors cannot be easily identified and a sensible dissection in convex subspaces is not obvious. Christopher Alexander's diagram of the complex field a point generates on a sheet of paper could be considered a precursor of such map, even if his analytical interest serves a different purpose than my own (Alexander 2002, p. 82). I propose a map based on a sequence of isodistant-lines from a point location (Figure 2). Since this map has a certain visual similarity to fingerprints and since it illustrates the metric uniqueness of any point location in a plan, I suggest calling it the Metric Fingerprint (MF).

The increase of metric distance from a point location could also be mapped with a continuous gradient, but isodistant lines provide additional data and are combined more easily with other layers of information. Three things speak for using isodistant lines over a continuous gradient in particular. First, isodistant lines are readable as such even when overlaid with isovist fields or other two dimensional plan colorings. Second, isodistant lines reveal a system of changing centers for isodistant-lines that do not show up clearly in a gradient. Isodistant-lines can display different areas of concentricity as distinct concentric regions in a plan system, with each region referring to the center of its respective concentric circles. Third, gradient representation of distance obscures an important feature of polygons that are topologically not simply connected. In such polygons, isodistant-lines intersect to form fissures or "watersheds" behind holes in the plan. These watersheds are curves that describe the position of all points where the distance to the viewing point around the whole or obstacle is exactly equal in both directions. That is to say: watersheds describe the position of all points where lines of equal distance intersect behind a hole. Since there are always at least two different centers for isodistant-lines behind a two-dimensional hole, every hole in a plan contains intersecting isolines and watersheds. For this reason, the number of watersheds in a plan indicates its topological order. In systems with more than one hole, watersheds occasionally intersect with each other to form veritable networks. This rather surprising phenomenon makes it impossible to predict directions of shortest paths behind holes without calculating the watersheds (Figure 3).

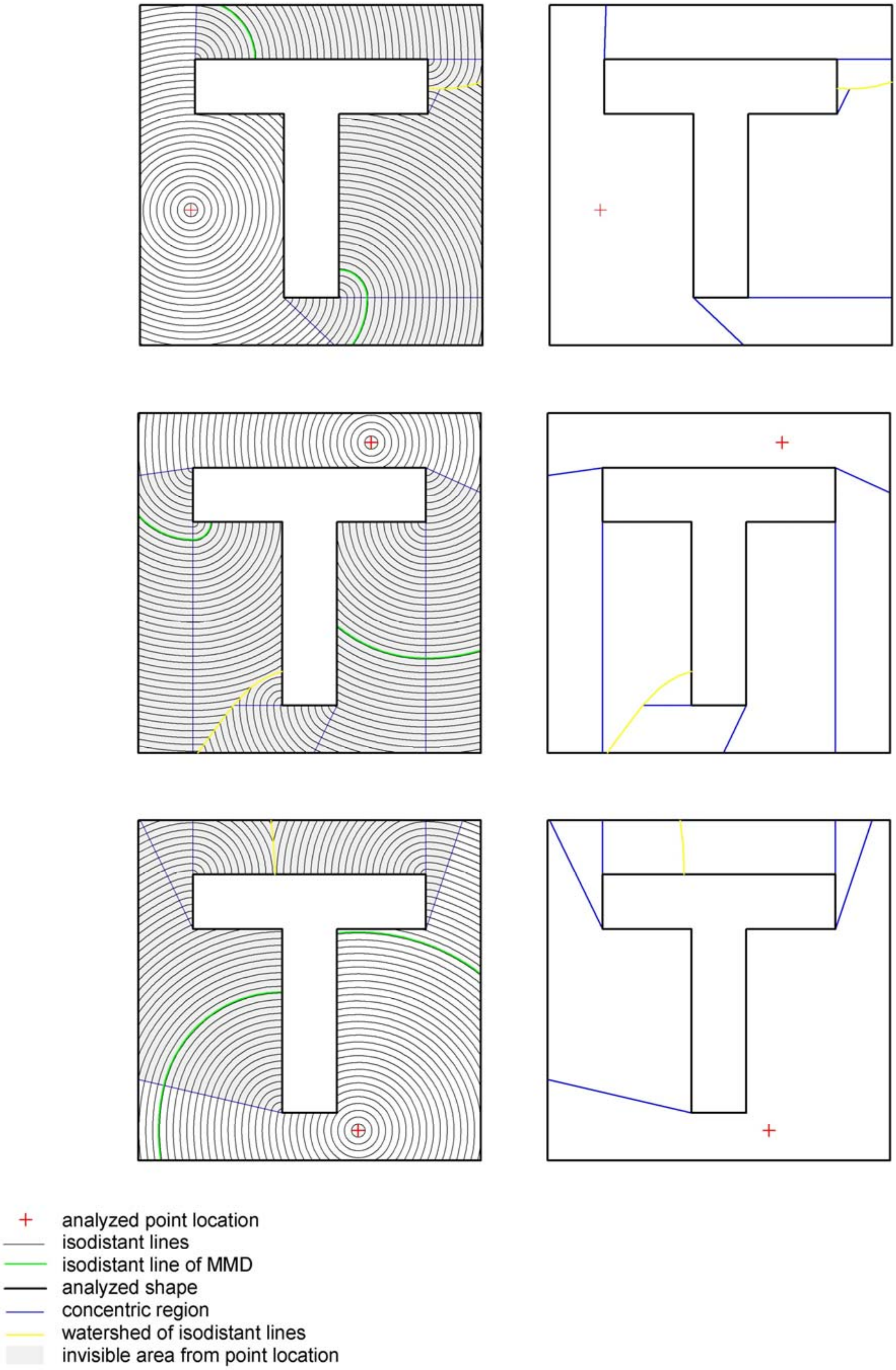


Figure 2
Metric Fingerprints and Concentric Regions

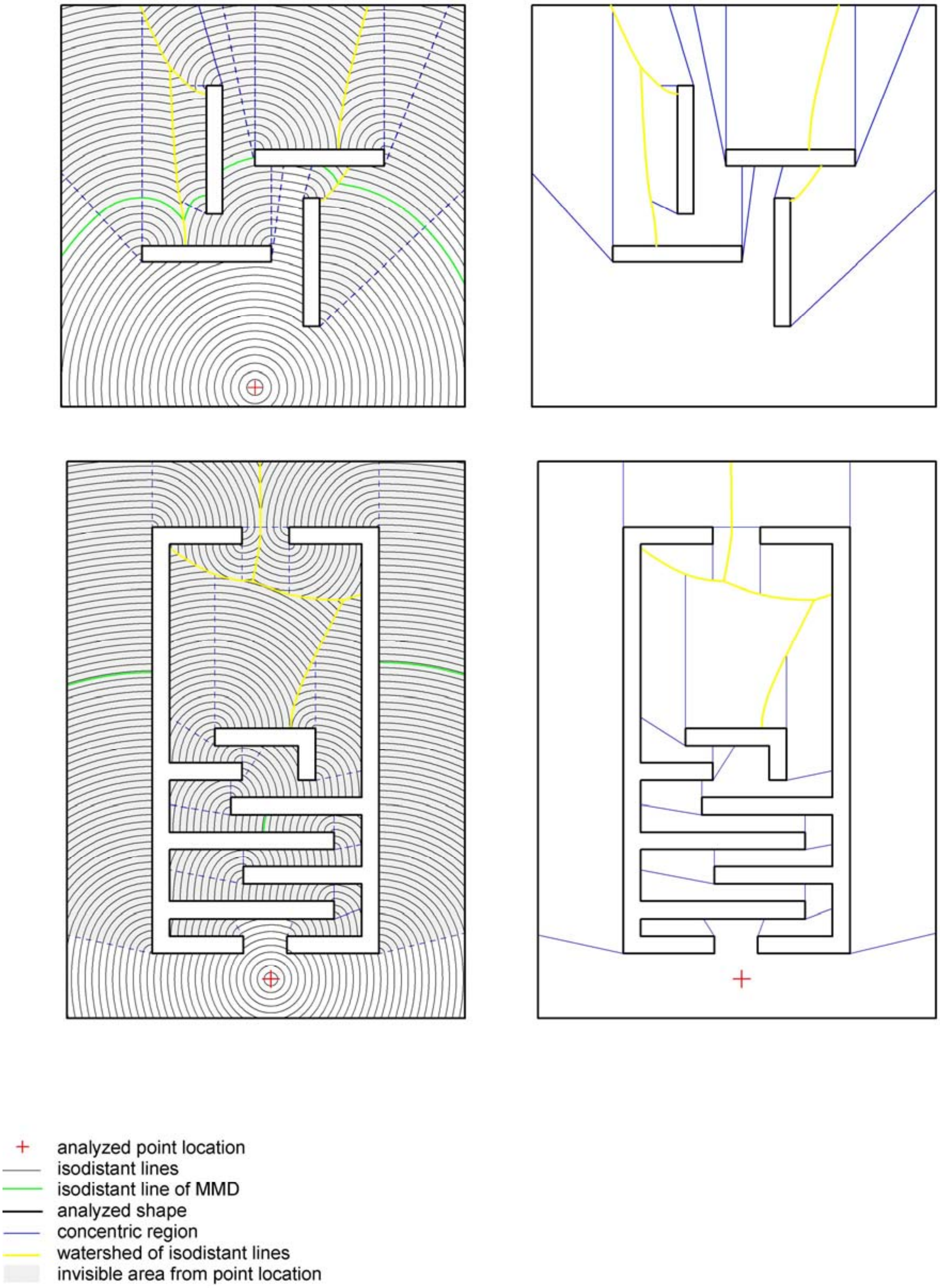


Figure 3
Examples of Intersecting Watersheds

Since watersheds represent lines of demarcation of directional change for shortest paths around holes, they have potential application for wayfinding. It would be interesting to examine, for instance, to what extent people are able to correctly identify watersheds in an urban block system, or to what degree assumptions about the shortest path around an obstacle influences movement. Worthy of note in this context is the fact that the detours made when crossing a watershed are always longer from a point nearby the hole than from a point far from the hole. In other words: the ripples that a hole or an obstacle creates in a Metric Fingerprint flatten out with increasing distance from the hole. This reduction of detours around the hole with increasing distance can be regarded as a mathematical argument for Hillier's assumption, that the "impact of objects on metric structure is localised compared with the effects of visual structure, while the metric structure of the large scale system of space is little affected by local metric variations." (Hillier 2007, pp. 001.10-11, Figure 4).

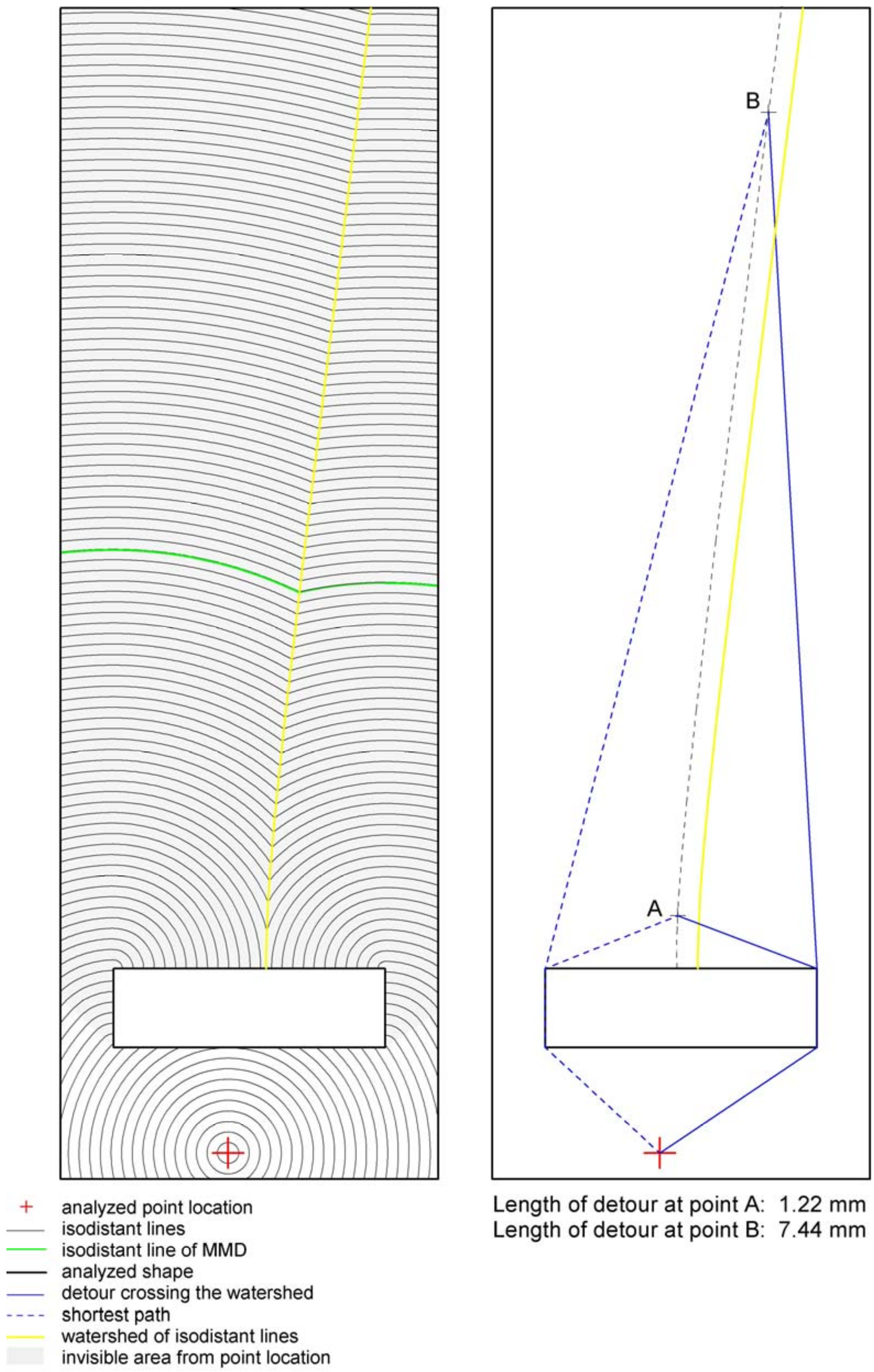


Figure 4

Flattening of Metric Ripples and Dimunition of Detours Behind a Hole

Let me summarize the main aspects of our model. A Metric Fingerprint consists of

1. The marking of a point location in a plan.
2. Isodistant lines of the point location (measured around angles).
3. The notation of watersheds of isodistant lines behind holes.
4. The optional notation of critical distances (MMD, maximum distance).
5. Other layers of information (isovist field, 2d colouring, captions).

3. Metric Fingerprints of the Farnsworth House

In exploring the analytical potential of metric fingerprints, I applied our model to Mies van der Rohe's Farnsworth House. This choice was motivated by my interest in the access-structure of open-plan buildings. The access-graph - the usual method in Space Syntax for describing the access-structure of housing plans - fails in open-plan buildings because of its lack of discrete spatial elements, like walled rooms and doors, that could be abstracted as nodes of a connected graph. One way to save the access graph would be to find a plausible dissection of Mies van der Rohe's space into convex subunits. But in my view a convex breakdown of the open plan would neglect the architectural concern for spatial continuity and overlapping regions that generally motivates open-plan arrangements.

The Metric Fingerprint for the entrance to the Farnsworth House shows a clear gradient of privacy along metric distance (Figure 5). Closest to the entrance is the eating place. Equally far from the entrance are the kitchen and the chair ensemble located in front of the fire place. Not surprisingly, the farthest place from the entrance is the bed, located behind the service core. What is astonishing about the position of the bed is its asymmetry with respect to the wall of the service core and the fact that both routes from the entrance around the service-core to the bed are equally long even though the core has an asymmetric position and the entrance has a symmetrical position. This equal length of routes around the core wouldn't have been the case if Mies had placed the bed symmetrically in relation to the core's wall. Hence, the asymmetry of the bed's position serves as a subtle metric strategy to move the bed as far as possible from the entrance while creating equal alternative routes though the kitchen and living spaces. As an additional feature, the asymmetry facilitates the positioning of the wall unit. The watersheds in Mies's plan illustrate how the use of open space with free-standing furniture can generate alternative routes and a choice of paths even in a relatively small house. By indicating the topological order of the plan, these watersheds reveal the "ringiness" of the access structure, which in the traditional access-graph is expressed as the cyclomatic number of the graph (Steadman 1983).

To grasp the flow of space and the dynamic change of locations in space in the Farnsworth House, I made films of Metric Fingerprints by rendering series of single Fingerprints along continuous trajectories through space and playing them as a sequence. Films of Metric Fingerprints produce space-time representations of the dynamic flow of the metric configuration (Penn 2003). They show that every point in space has indeed its unique metric fingerprint, its own metric genius loci. Moving through space metric change is smooth and occurs continuously. But a film displaying only the change in concentric regions and watersheds without isodistant lines shows that continuous trajectories generate discontinuous and abrupt change in the configuration of concentric regions (Figure 5). At the level of concentric regions, space becomes syntactic, discontinuous and nonlinear. Given that concentric regions are substructures of the metric field, the question is whether the observed discontinuity in their configuration is a mere product of abstraction, or if it is a real phenomenon with further practical significance.

Calculating metric mean distance with triangulated metric regions The different concentric regions of a Metric Fingerprint can be constructed without drawing sequences of isodistant lines, since the shape of concentric regions shows that they are just a series of subsequent, non-overlapping, truncated isovists. In order to draw this series of isovists efficiently, I propose the following procedure implemented as a CAD-program. First, an isovist of the point location of interest must be constructed. Then, all "obstructing angles" of the isovist, ie. angles that hinder the view in other regions of the plan are identified. After the isovists of all obstructing angles are drawn, the primary

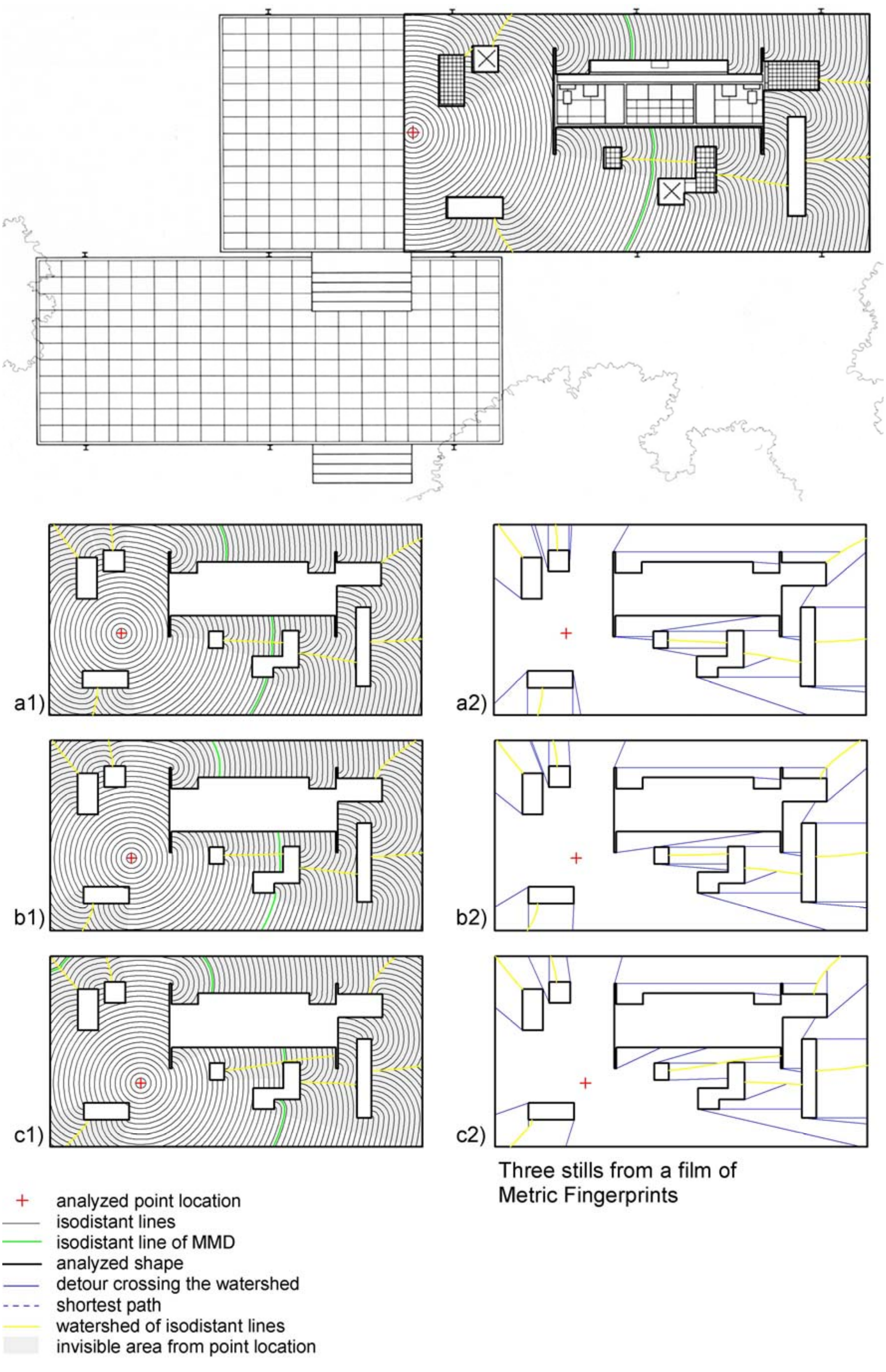


Figure 5
Metric Fingerprints and Concentric Regions of the Farnsworth House

isovist is subtracted from the isovists of the obstructing angles. If the secondary isovists still have obstructing angles that hinder the view in other regions of the plan, the procedure is repeated until all isovists cover the surface. Subsequent isovists must be truncated with the preceding ones on the shortest path back to the original viewing point. In the case of surfaces that are not simply connected, an additional step is needed because there will always be at least two overlapping isovists behind any hole of the plan system. The regions of these isovists have to be separated along the above mentioned watershed, which represents all points equidistant to the original viewing point both ways around the hole. On the right of the watershed, the shortest paths around the hole are directed to the right; on the left of the watershed, they are directed to the left. It should be noted that watersheds may intersect with each other, causing both trajectories to merge behind the intersection point. For this reason, watersheds have to be constructed before any isovists behind a hole can be split.

This dissection of the plan into concentric regions of subsequent non-overlapping isovists has a very useful feature: the shortest paths of all points in a concentric region pass through the viewing point of the isovist of that region. This means that all points in every concentric region differ in their metric distance to the original point location only insofar as their position is different from the viewing point of the isovist they belong to. Up to this viewing point, their shortest paths are equal. This allows for a radical simplification in calculating MMD: If the polygonal concentric region is triangulated - and if we take the viewing point of the truncated isovist always as corner A of the triangle - the problem of calculating metric mean distance of a point to a general polygonal surface is reduced to calculating the mean distance of all points of a surface of a triangle to point A of the triangle. And the latter I have solved by means of mathematical integration.[1] Once the mean distance of all triangles to their respective point A is known, one only has to add the distance point A has from the original viewing point to the mean distance of A to the triangle. The mean distance of the viewing point to the entire plan system can then simply be derived as a surface-weighted mean of the mean distances of all triangles from the viewing point.

The results of this continuous model of metric mean distance are in accordance with values obtained with the discrete graph model of MMD. Unfortunately, the continuous model has not yet improved computing time. This is partly due to the fact that the dissection of the plan into concentric regions involves a time-consuming shortest-path algorithm for all angles of the analysed polygon. It is also partly due to the implementation of the algorithm in a CAD package that has proved very practical from an architectural point of view, but in terms of numerical speed still needs improvement.

4. Statistics of Centrality

MMD provides a value of mean shortest paths in plans. As such, it can predict the expected movement effort from a certain location if real routes can be approximated by shortest paths and if the endpoints or starting-points of routes can be approximated as homogenously distributed over the surface of a plan. Unfortunately, these two conditions are not always fulfilled. And they cannot simply be assumed about human movement patterns since humans neither stick to shortest paths nor are the starting- or endpoints of their routes in general homogenously distributed over a plan. Still, there may be special cases where both conditions of MMD may apply even to human movement. A trivial example would be the allocation of a fixed telephone in a housing plan. In this case, one could use MMD to minimize the average effort to reach the phone. Here, the assumption that people like to take the shortest path if the phone rings and that their positions at the moment the phone starts to ring are homogenously distributed over the plan might be realistic.

In other cases, it would actually be much more reasonable to assume that starting- and endpoints of human movement are distributed along the edge rather than the surface of a plan because points of interest in a plan system tend to be concentrated on the edges. A good example for this phenomenon of the edge as an attractor is the position of furniture in housing plans or the position of entrances and shops at the edges of public squares. If the edge is so important as a starting or end-point of routes, Mean Edge Distance (MED) might provide a more realistic model than MMD for modelling the average effort of movement.

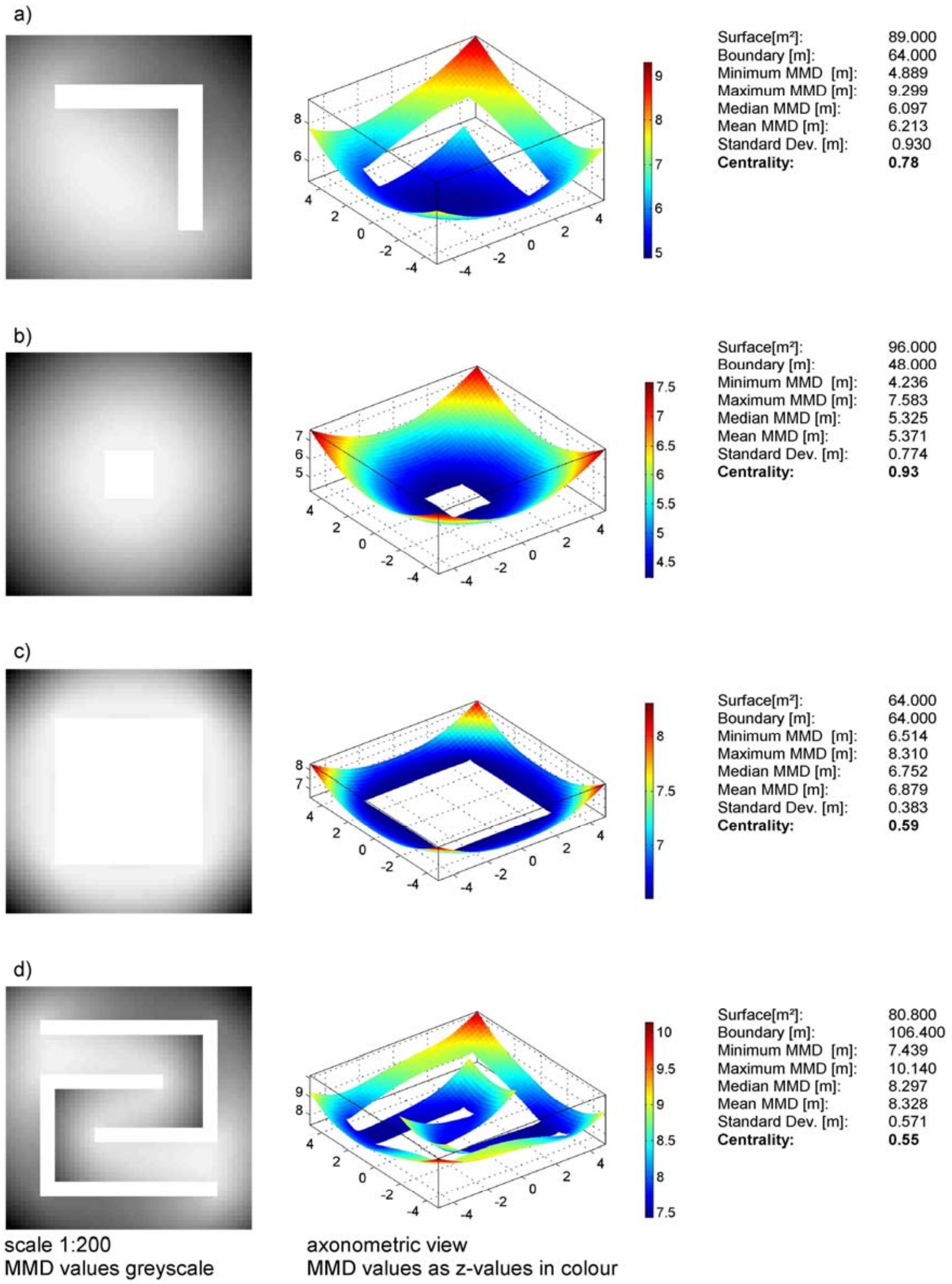


Figure 6

Metric Centrality of Shapes, Statistics of MMD Values, Centrality

Apart from representing the effort of average movement in a few special cases, MMD can be used as a statistical distribution to describe the centrality of plans. So far, the architectural concept of centrality is based on the existence of an axis of symmetry. The center in a plan is given either by an intersection of at least two axes of symmetry or by the midpoint of one axis. If there is no axis of symmetry, however, we can't point to a center in the classical sense, even if it is intuitively quite clear that an L-shaped plan has regions that are metrically more central and others that are metrically more peripheral. MMD can be considered an analytically derived and mathematically precise value of this intuitive perception of more peripheral and more central locations in asymmetric plans. Taking MMD values of all point locations as indicators of centrality, we can replace the center of a plan with a gradient of relative centrality. It is remarkable that the distribution of MMD values reliably has its minimum in symmetric shapes at the point of the classical center. The new notion of centrality as the minimum average shortest path to all points in a plan thus includes the classical notion of the center as a point that is always identified. Yet at the same time this new notion of centrality provides a more general model for defining centers than the one based on axes of symmetry, and thus extends the notion of centrality to asymmetric shapes. The notion that the center is a minimum in a field of MMD values also implies that shapes can have multiple centers, since it is not unusual for shapes with holes to have more than one local minimum.

Moreover, the centrality of a shape in a plan can be easily expressed in a term that is independent of the size of the shape by taking the mean of all MMD-values of a shape and normalising it with the respective mean value of a circle having the same surface area. This way, the overall centrality of plans becomes quantifiable and comparable independently of shape size. A sample of values C of Centrality for a set of rectangular shapes is given in Figure 6.[2]

We also suggest that the range of MMD-values in a surface be taken as an object of statistical investigation. Maximum, Minimum, Mean, Median and Standard Deviation can be used to characterise the distribution and spread of MMD values in a plan. A systematic survey of these statistical values in plans - understood as an allometry - might yield typological insight about housing plans or urban spaces. It would, for instance, be interesting to find out whether certain housing types fit into a relatively narrow bandwith of MMD values and distributions and whether simple relationships between the size of a plan and the distribution of its MMD-values are the same as those demonstrated for building volumentry and floor space (Steadmann 2006, Bon 1973).

Since no allometric studies on the basis of MMD-values have been published so far, we must make their case using a plausible conjecture. Since average room size in housing plans generally doesn't scale linearly with floor area - that is to say, bigger apartments tend to have more rooms rather than just an equal amount of bigger rooms - and because we know that there is an upper limit on depth in housing plans due to day-light issues, we can likely expect that in a sample of housing plans the mean MMD value increases by a factor that is greater than the increase of floor area would suggest. If we could determine this factor, we could arrive at a morphological rule about the relationship of spatial differentiation and floor-area in housing plans.

5. Conclusion and further methodological research

I have argued that it is possible to analyse the metric properties of a plan without relying on the abstraction of a finite set of discrete spatial elements, as in graph-based models of Metric Mean Distance. This led me to propose a new analytical map, the Metric Fingerprint, and to identify a system of concentric regions in the metric field that represents its discrete substructure and allows for a continuous calculation of MMD.

Since my present implementation of the new algorithm for MMD in a CAD package hasn't improved the speed of calculation compared to the graph-based model, I propose improving computing time by porting the algorithm to a powerful programming environment for numerics, such as MATLAB. Implementing a slightly modified algorithm that calculates Mean Edge Distance also seems to be of further interest and requires only minor changes to the program.

I am also considering calculating Visual Integration with a continuous model based on non-overlapping subsequent isovists. In order to do that, a dissection of regions in the plan is needed that exactly matches the distribution of pixels with equal visual step depth in visual integration. It can be shown that progressive isovists generate this kind of a dissection if they are constructed as isovists of the entire edge of the preceding isovist and not as isovists from the point of an obstructing angle, as is the case with MMD (Figure 7). Although the key-question of how to represent visual step depth in a graph with continuous surfaces has already been resolved, the algorithmic details have yet to be worked out.

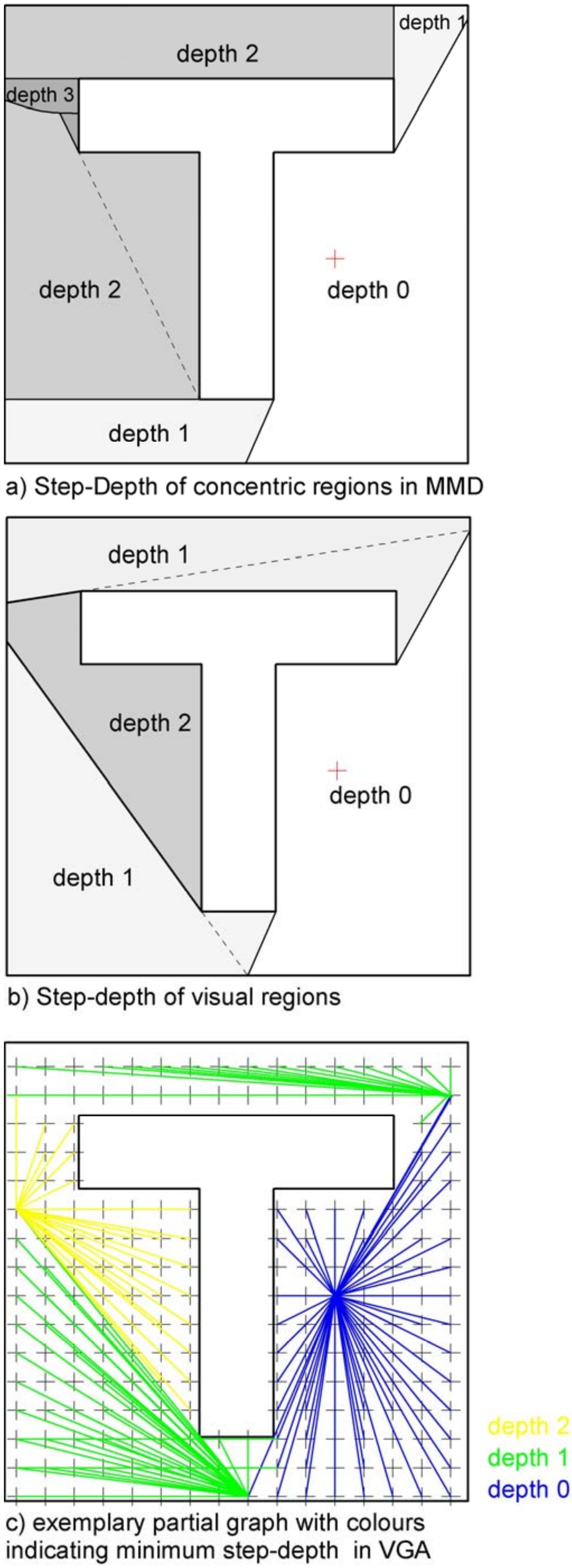
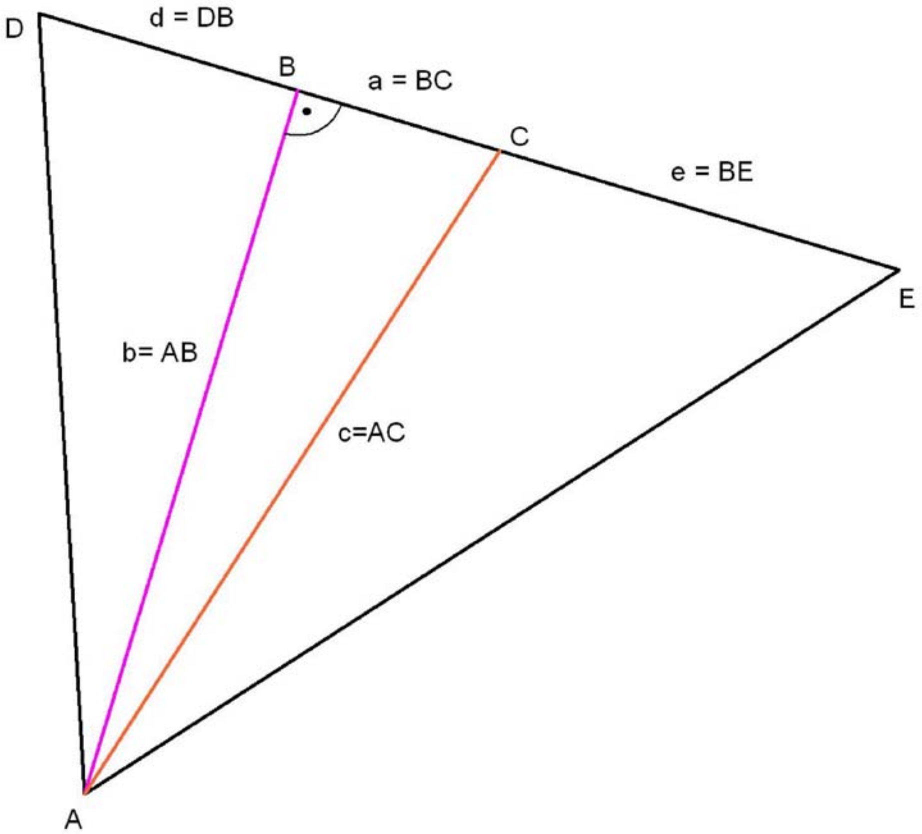


Figure 7

Comparison of Surfaces of Equal Step-Depth in Visual Integration and Metric Mean Distance



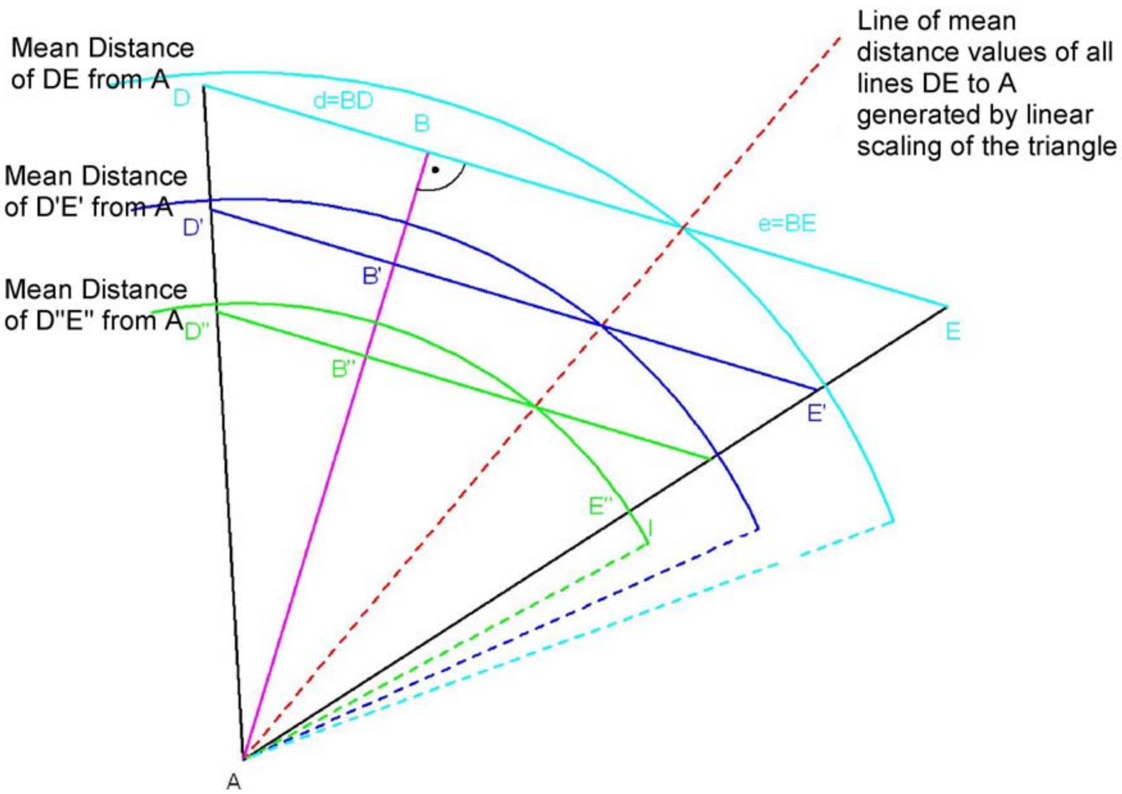


Figure 8

The Metric Meandistance of a Corner to the Surface of a Triangle

Notes

1 The Metric Mean Distance of a point A to a triangle is calculated by integration in two steps (Figure 8). First we calculate the mean distance of the opposite line from A to A. The distance of any point C on line DE is given by:

$c = \sqrt{a^2 + b^2}$ (Pythagoras' Theorem)

The sum G of distances of all points on DE from A thus yields the integral:

$$G = \int_d^e \sqrt{(a^2+b^2)}da = \left[1/2 * \left(\ln\left(a + \sqrt{(b^2+a^2)}\right) * b^2 + a * \sqrt{(b^2+a^2)}\right)\right]_d^e$$

The mean distance M of line DE from A is then given by:

$$M = G/(d+e)$$

Fortunately, the Integral of G scales linearly (see Figure 8b) since:

$$G/k = \left[1/2 * \left(\ln\left((a/k) + \sqrt{((b/k)^2 + (a/k)^2)}\right) * (b/k)^2 + (a/k) * \sqrt{((b/k)^2 + (a/k)^2)}\right)\right]_{(d/k)}^{(e/k)}$$

This implies that a linear scaling of the triangle at point A creates a line of mean values of DE from A. Integrating these mean values along the line is the second step of integration. The total distance D of all points in the triangle from A is therefore expressed by the integral:

$$D = \int_0^b M * (d+e)/b^2 * x^2 = \left[M * (d+e)/b^2 * x^3/3\right]_0^b = M * (d+e) * b/3$$

Metric Mean Distance can then be derived by dividing D by the surface of the triangle:

$$M_{md} = (M * (d+e) * b/3)/((d+e) * b/2) = M * 2/3$$

expressed alternatively as:

$$M_{md} = 2/3 * \int_d^e \sqrt{(a^2+b^2)}da/(d+e) = 2/3 * \left[1/2 * \left(\ln\left(a + \sqrt{(b^2+a^2)}\right) * b^2 + a * \sqrt{(b^2+a^2)}\right)\right]_d^e/(d+e)$$

2 The formula for calculating values of centrality is derived as follows: If MMMDs is the mean value of all MMD values in a plan shape s and MMMDc is the mean value of all MMD values in a circle c with the same surface as shape s, then a size-independent measure C of centrality for surface s is given by: $C = MMMD_c / MMMD_s$

The reason behind a normalisation using the values of a circle as reference is that the circle is the most central arrangement of points and therefore it is also the shape with the absolute minimum MMMD in respect to a particular surface area. Moreover, MMMD values of circles with particular sizes can be easily calculated from one reference size, as MMMD values scale linearly with shape diameter (though not with surface size!) Values obtained this way for Cs range from 0 to 1 with 0 indicating an infinite value of MMMDs and 1 indicating that s is also a circle.

References

Alexander, Christopher. 2002. *The Nature of Order*, *Book 1*, *The phenomenon of Life.* Berkeley*:* Center of Environmental Structure. 79-108.

Bon, Ranko. 1973. Allometry in the Topologic Structure of Architectural Spatial Systems. *Ekistics.* 36 (215). 270-276

Hillier, Bill et al.. 2007. Metric and Topo-Geometric Properties of Urban Street Networks: Some Convergences, Divergences and New Results. *Proceedings, 6th International Space Syntax Symposium.* 001.01-22

Hillier, Bill and Julienne Hanson. 1984. *The Social Logic of Space.* Cambridge: Cambridge University Press.

Hillier, Bill. 1996. *Space is the Machine.* Cambridge: Cambridge University Press.

Hillier, Bill. 1999. Centrality as a Process Accounting for Attraction Inequalities in Deformed Grids. *Urban Design International.* 3/4. 107-127.

Montello, Daniel R. 2007. The Contribution of Space Syntax to a Comprehensive Theory of Environmental Psychology. *Proceedings, 6th International Space Syntax Symposium.* IV.01-11.

Penn, Alan. 2003. The shape of habitable space. Proceedings, *4th International Space Syntax Symposium.* 62.1-62.16

Peponis, John et al.. 1997. On the description of shape and spatial configuration inside buildings: convex partitions and their local properties. *Environment and Planning (B).* 24. 761-781

Ratti, Carlo. 2004. Space Syntax: Some Inconsistencies. *Environment and Planning B.* 31. 487-499.

Steadman, Philip. 1983. *Architectural Morphology.* London: Pion.

Steadman, Philip. 2006. "Allometry and Built Form: Revisiting Ranko Bon´s Work with the Harvard Philomorphs. *Construction Management and Economics.* 24. 755-765